A Multi-Scale Hybrid Approach for Modeling Respiratory Tractable Infectious Diseases

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Introduction

The modeling of respiratory-transmitted infectious diseases is often fragmented, focusing on specific aspects such as immune response, contact dynamics, or infection dynamics. However, for a comprehensive understanding of this phenomenon, it is essential to overcome the limitations of different modeling scales and assumptions. An integrated approach is necessary to bring together the various facets coherently within a well-defined conceptual framework. This approach, which must take into account the inherent complexity of modeling emerging respiratory-transmitted infectious diseases, should combine these multiple perspectives to obtain a more comprehensive and global view of the spread of these diseases.

In this poster, we present a hybrid, multi-scale, and spatio-temporal approach aimed at enhancing our understanding of the infection dynamics of emerging respiratory-transmitted infectious diseases. Our conceptual framework integrates social contact dynamics, allowing us to model the movement of individuals considered as virus vectors and their social interactions in various activity locations. It also considers the spatio-temporal concentration of the virus on surfaces and the dose of virus inhaled by each individual.

Methodology

1 Estimation of social contacts

An improved social force model is adjusted to simulate the contact dynamics among agents, incorporating the socio-cultural and demographic characteristics of populations across diverse social contexts. We consider a population of n agents, where the i-th agent is represented as a disk with center x_i, radius r_i, and mass m_i. Their interaction with the environment is governed by the so-called social forces.

2 Infection process

Airborne transmission, the primary route for spreading diseases like Influenza, Covid-19, Measles, and MERS, involves particles that can travel long distances or fall quickly to the ground. Our model covers both short-range and long-range transmission, simulating virus diffusion post-cough or sneeze. Eq.(2) incorporates environmental and seasonal variations by adjusting the diffusion coefficient with real-world data or calibration. Eq.(3) describes the inhaled viral load through direct or indirect transmission, and finally, the agent is classified as infected according to the probability given by Eq.(4).

Model

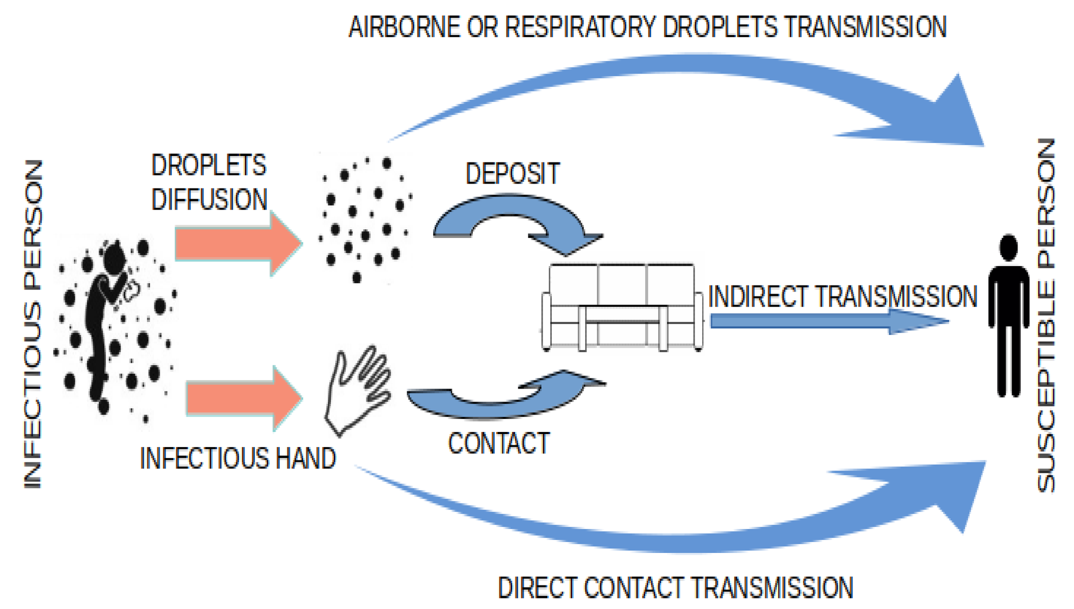
$$m_i \frac{d\vec{v}_i}{dt} = \vec{F}_{\text{rep},i} + \vec{F}_{\text{soc},i} + \vec{F}_{\text{int},i} + \vec{F}_{\text{ext},i} \qquad \text{(Motion equation)} \quad (1)$$

$$\frac{\partial C_a}{\partial t} = D\Delta C_a + \alpha W_d - \mu C_a \qquad \text{(Virus diffusion)} \quad (2)$$

$$\frac{dC_{acc,i}}{dt} = \rho C_a - \gamma C_{acc,i} \qquad \text{(Virus accumulation)} \quad (3)$$

$$P_{a,i} = 1 - \exp(-C_{acc,i} I) \qquad \text{(Infection probability)} \quad (4)$$

Where C_a is the concentration of the virus at a specific position x and at time t, $C_{acc,i}$ indicates the virus accumulated in the respiratory tract, and W_d represents virus production and spread on the mesh after a cough.



Routes of transmission of the different diseases. However, most of the diseases transmit mainly through airborne particles.

Some Results

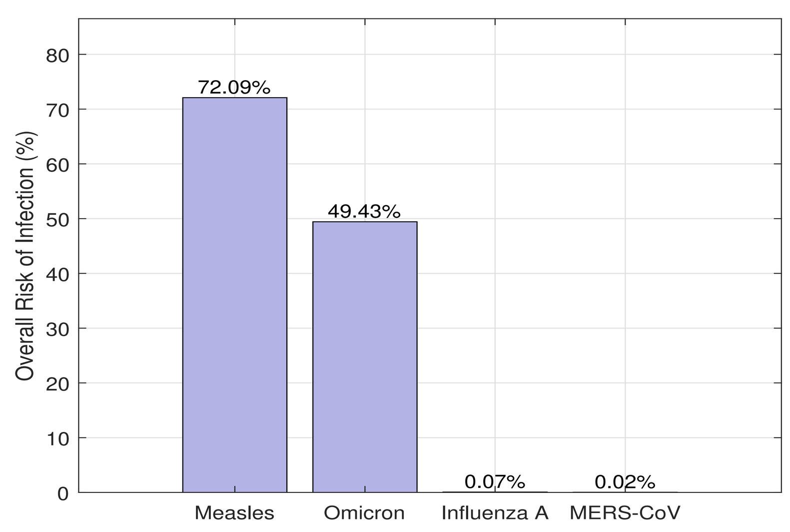


Figure 1: Average overall risk of infection across places of activity

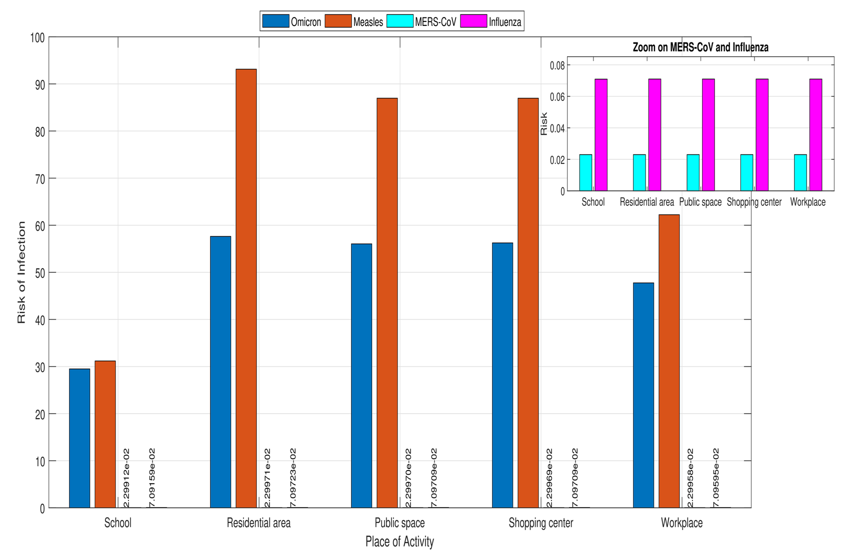


Figure 2: Risk of infection by Virus and place of activity

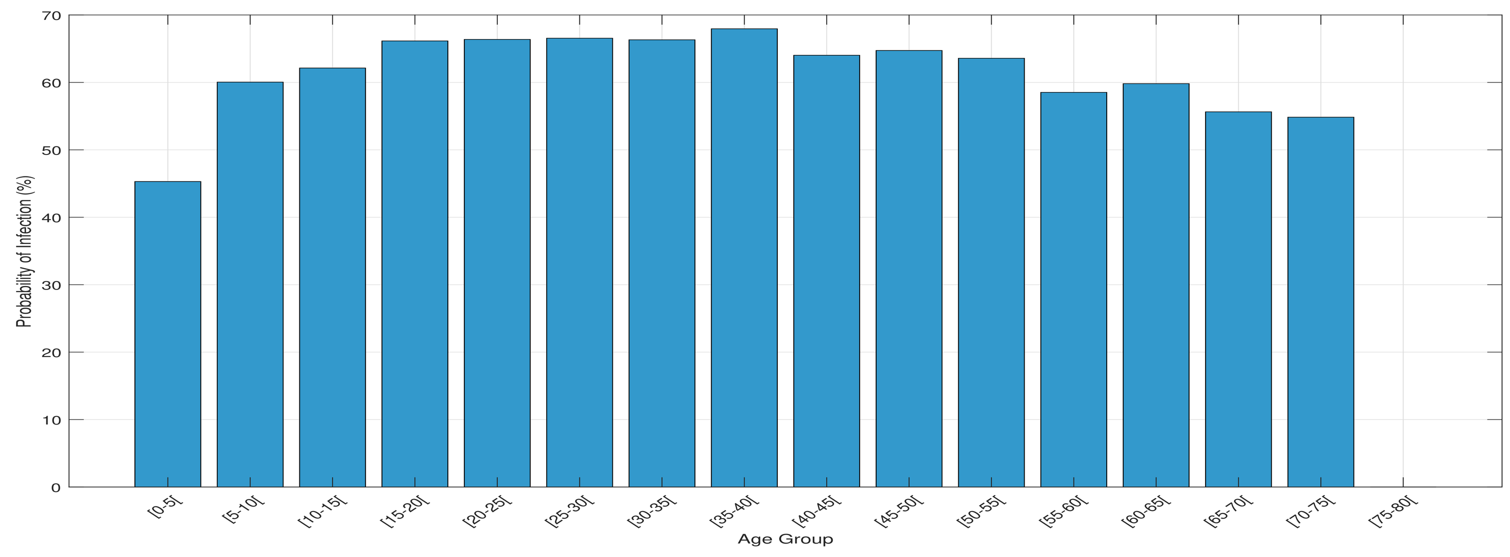
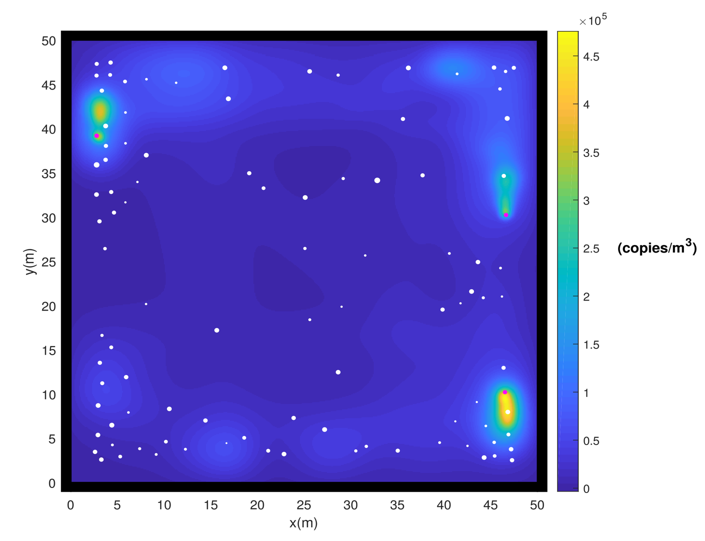


Figure 3: Probability of infection by Omicron variant in residential area

Setting of Numerical Simulations



A numerical simulation screenshot showing pedestrian locations and virus concentration in the air. White circles describe susceptible individuals. Pink ones represent infectious individuals, respectively. The size of each individual correlates with its weight. The green to yellow gradient describes the concentration of the virus in the air.

Conclusion and future Work

Key points Measles and Omicron constitute a greater threat to public health than other diseases. The risk of infection is higher in residential areas. **Further Insights:** Individuals in the range of 20–40 years old are more likely to be contaminated. Due to the chosen duration of the simulations, the present work does not highlight the response of the immune system. Future work will focus on macroscopic-scale modeling of a community by integrating flows between activity locations, as well as on the cellular scale in terms of immune response.

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Asymptotic behavior of the positive global solution to a stochastic mod

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Introduction

Mathematical models have become important tools in analyzing the spread and control of infectious diseases, we usually divide the population into the susptible S, the infective I, and the recovered R, and consider the following SIR model:

$$\begin{cases} \dot{S} &= b - \beta SI - \mu S, \\ \dot{I} &= \beta SI - \gamma I - \mu I - \alpha I, \\ \dot{R} &= \gamma I - \mu R. \end{cases} \tag{1}$$

So The deterministic approach has some limitations in the mathematical modeling transmission of an infectious disease and it is quite difficult to predict the future dynamics of the system accurately.

Stochastic differential equation models play a significant role in various branches of applied sciences including infectious dynamics, as they provide some additional degree of realism compared to their deterministic counterpart. Also, Deterministic models do not incorporate the effect of fluctuating environment.

The stochastic model:

$$\begin{cases} dS(t) &= [b - \beta S(t)I(t) - \mu S(t)]dt - \sigma S(t)I(t)dB(t), \\ dI(t) &= [\beta S(t)I(t) - \gamma I(t) - \mu I(t) - \alpha I(t)]dt + \sigma S(t)I(t)dB(t), \\ dR(t) &= \gamma I(t) - \mu R(t). \end{cases} \tag{2}$$

Problematic

we consider the effect of stochastic fluctuations of environment to the endemic equilibrium of the corresponding deterministic system and we compare the asymptotic behavior of E_* of deterministic system (1) and stochastic system (2), addition to bahavior of E_0 of system (2) .

Key Results

1. The stochastically asymptotic stability in the large of the disease-free equilibrium when $R_1 < 1$ $\left(R_1 = \dfrac{\beta b/\mu + (b/\mu)^2\sigma^2/2}{\gamma + \mu + \alpha}\right)$.
2. There exists an endemic equilibrium E_* of system (1) when $R_0 > 1$, but system (2) no longer has any endemic equilibrium of (1). We obtain
$$\lim_{t\to\infty} sup \frac{1}{t}\int_0^t \left[(S(u)-S^*)^2 + (I(u)-I^*)^2 + (R(u)-R^*)^2\right] du \le K\sigma^2$$
a.s. if $R_0 > 1$, where K is a positive constant.

Method

1. Existence and Uniqueness of the Nonegative solution
2. Asymptotic Behavior of the Disease-Free Equilibrium
3. Asymptotic Behavior Around the Endemic Equilibrium of the Deterministic Model
4. Numerical simulation

Examples and plots

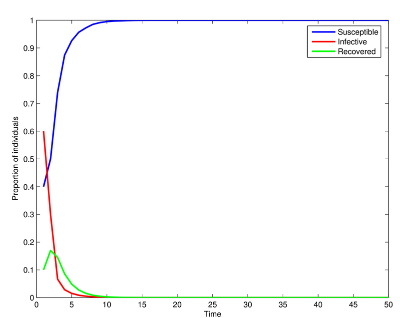

Fig. 1: The convergence of the solutions to the disease-free equilibrium E_0

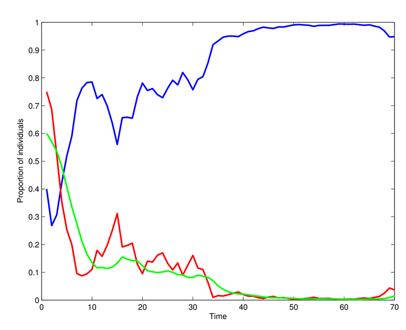

Fig. 2: The convergence of the solutions around the endemic equilibrium E^*

Conclusion and future Work

After analyzing our suggested model, we found that the system (2) has only one global positive solution, using stochastic Lyapunov functional methods, we deduce the free equilibrium global asympotical stability and exponential meansquare stability under some conditions, Moreover we explore the asymptotic behavior of the solution around the endemic equilibrium of the deterministic model. Our analysis suggests that future research should look into various fascinating issues, such as incorporating our model with an optimal control problem and offering additional random models to different phenomena.

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LA CONSOLIDATION UNIDIMENSIONNELLE DES DIFFERENTS TYPES DES SOLS

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Introduction

Depuis que Terzaghi a publié sa théorie de la consolidation, les travaux de recherche sur les problèmes de consolidation se sont fortement multipliés. Les théories de la consolidation ont souvent négligé la non-linéarité du sol pour les fins. Et Depuis que Davis et Raymond [1] ont proposé pour la première fois la théorie de la consolidation non linéaire basée sur l'hypothèse que la diminution de la perméabilité est proportionnelle à la diminution de la compressibilité au cours du processus de consolidation et que la contrainte effective initiale est constante avec la profondeur, de nombreuses tentatives ont eu lieu pour développer différents modèles de consolidation unidimensionnels prenant en compte les variations non linéaires de perméabilité et de compressibilité.

Problematic

La théorie de la consolidation unidimensionnelle primaire de Terzaghi est une théorie qui prédit la variation de pression interstitielle à l'intérieur d'un système compressible chargé. La variation de la pression interstitielle se traduit par une variation de contrainte effective, elle-même commandant à son tour le tassement du système chargé. Cette théorie nécessite les hypothèses de base suivantes :

- **Le sol est complétement saturé d'eau,**
- **Les grains constituant le sol et l'eau contenue dans les pores sont incompressibles,**
- **Les fluides suivent la loi de Darcy,**
- **Les déformations du système solide dépendent exclusivement des contraintes effectives par une relation linéaire indépendante du temps,**
- **Le squelette solide est homogène, c-à-d que les relations contrainte-déformation et vitesse-gradient de pression sont indépendantes de la profondeur,**
- **Les variations des déformations, des vitesses et des contraintes sont faibles et de plus la théorie est quasi-statique.**

L'équation de la consolidation unidimensionnelle qui exprime la variation de la pression interstitielle avec la profondeur en fonction de sa variation dans le temps et le chargement appliqué, peut être résumé pour un sol multicouche sous l'effet d'un chargement qui dépend du temps comme suit :

$$\boldsymbol{C_v}^{i}\frac{\partial^2 \boldsymbol{u_i}}{\partial \boldsymbol{z}^2}=\frac{\partial \boldsymbol{u_i}}{\partial \boldsymbol{t}}-\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{t}} \qquad (1)$$

Method

- Excès de la pression d'eau interstitielle :

$$\frac{\partial u^i}{\partial t}=C_v{}^{i}\left[\frac{\partial^2 u_i(z,t)}{\partial z^2}+\frac{1}{\sigma'^{i}(z,t)}\left(\frac{\partial u_i}{\partial z}\right)^2\right]+\frac{\partial q}{\partial t} \quad ;\ i=\{1,2,\ldots\ldots,n\} \qquad (2)$$

Après simplification en définissant une variable : $\omega_i(z,t)=\ln\left(\frac{\sigma'^{i}(z,t)}{\sigma'_0+q(t)}\right)$, devient :

$$\boldsymbol{u_i(z,t)=[\sigma'_0+q(t)]\left[1-e^{\omega_i(z,t)}\right]} \qquad (3)$$

- Degré moyen de la consolidation :

$$\boldsymbol{U_S^i=S=\frac{1}{lnN_Q}\left[\ln\left(\frac{\sigma'_0+q}{\sigma'_0}\right)+\frac{1}{h_i}\int_{z_{i-1}}^{z_i}\omega_i\,dz\right]} \qquad (4)$$

Examples and plots

Les figures ci-dessous présentent la variation de temps de consolidation avec le degré moyen de la consolidation pour différents types de sols (Kaolinites et Limons) qui est constitué par une seule couche d'un mètre d'épaisseur, en variant le paramètre de chargement Nq qui présente le rapport entre la contrainte effective final et initiale.

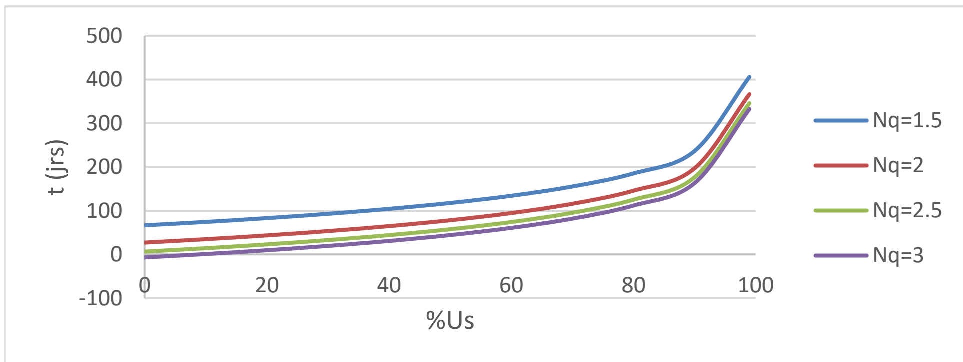

Figure 1 : *Variation du temps de consolidation en fonction de degré moyen de la consolidation des Kaolinites*

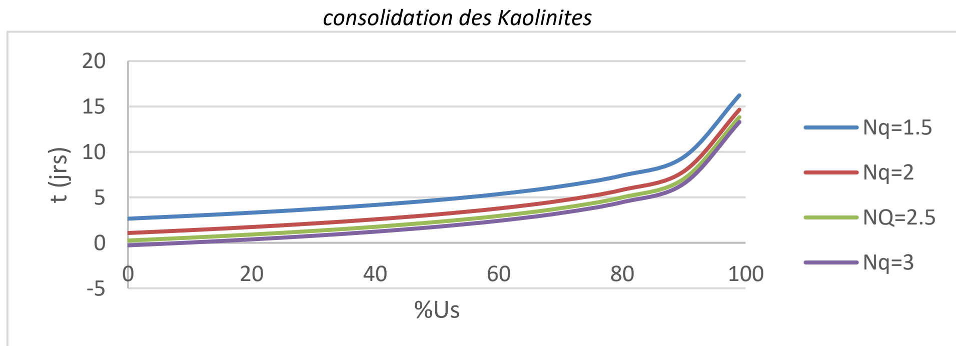

Figure 2 : *Variation du temps de consolidation en fonction de degré moyen de la consolidation de Limon*

Key Results

- Le coefficient de consolidation Cv présente un effet sur le temps de la consolidation qui est donné par :

$$\boldsymbol{t=\frac{T_v.H^2}{C_v}} \qquad \textbf{(5)}$$

Et puisque chaque type de sol présente un coefficient de sol différent, alors le temps de la consolidation variera d'un type à l'autre.

- Le gradient de la variation de degré moyen de consolidation en fonction de temps de consolidation se diffère selon le type de sol et surtout selon son coefficient de consolidation; tel que plus le coefficient de consolidation du sol est grand plus le temps de consolidation est court, par exemple pour arriver à un degré de consolidation de 50% les Montmorillonites nécessitent 58 jours pour un paramètre de chargement de valeur de 2.5, cependant le limon dont le coefficient de consolidation est le plus grand ne nécessite que deux jours.

Conclusion and future Work

Ce poster montre des solutions analytiques qui ont été dérivées pour la consolidation non linéaire unidimensionnelle d'un sol multicouche saturé sous un chargement dépendant du temps.
Il convient de noter que les solutions actuelles sont basées sur l'hypothèse que la distribution de la contrainte effective initiale est constante avec la profondeur et que le coefficient de consolidation de chaque couche ne varie pas avec le temps sous un chargement dépendant du temps.
D'après l'étude de comportement de la consolidation qui a été faite en se basant sur ces solutions proposées, on arrive à conclure que l'augmentation de coefficient de la consolidation est proportionnelle avec la diminution de temps de consolidation, et plus le paramètre de chargement Nq est grand plus la consolidation est rapide.

L'étude a été faite pour un sol à un seul drainage en attendant à approfondir l'étude en traitant un sol de double drainage.

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Mathematical modeling of e-commerce

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Introduction

In this research, a mathematical model $SAWPD$ was proposed, which is divided into five compartments. It is a non-linear deterministic mathematical model in order to study and analyze electronic commerce. This is also done using differential equations. As for the number of stores that oppose e-commerce, it is obtained using the P. Van Den approach. In this research, the necessary conditions were created to achieve local and global asymptotic stability for equilibria devoid of stores that oppose electronic commerce, as well as the necessary conditions for achieving local and global asymptotic stability for equilibria that include stores that oppose electronic commerce. After evaluating and analyzing the factors controlling the increase in the spread of e-commerce demand among shops, as well as the factors contributing to the spread of shops opposing e-commerce, Finally, we used the MATLAB application for numerical simulations that give us a comprehensive vision of the policies and initiatives that must be followed in order to reach the desired goal.[1] [2].

Mathematical modeling

Compartment S : Traditional shops that do not use electronic commerce.
Compartment A : Shops that oppose the use of electronic commerc.
Compartment W : Shops that use electronic commerce.
Compartment P: Shops that have temporarily stopped using electronic commerce.
Compartment D :Shops that were using e-commerce and stopped doing so permanently.
with $N(t)$ indicates the number of stores at time t such as

$$N(t) = S(t) + A(t) + W(t) + P(t) + D(t).$$

In order to represent the dynamics of the $SAWPD$ mathematical model, we relied on the following nonlinear system:

$$\begin{cases} \frac{dS}{dt} &= \Lambda - \frac{\beta SA}{N} - \alpha S - \mu S \\ \frac{dA}{dt} &= \frac{\beta SA}{N} - \gamma A - \mu A \\ \frac{dW}{dt} &= \alpha S + \gamma A + \theta P - \eta W - \delta W - \mu W \\ \frac{dP}{dt} &= \eta W - \theta P - \mu P \\ \frac{dD}{dt} &= \delta W - \mu D \end{cases} \quad (1)$$

With the initial conditions $S(0) \geq 0$, $A(0) \geq 0$, $W(0) \geq 0$, $P(0) \geq 0$ and $D(0) \geq 0$.

Parameter	Description
Λ :	The rate of new shops being added to old shops
α :	Percentage of shops that want to use electronic commerce.
γ :	The rate of stores that have converted to using e-commerce.
μ:	The rate of shops that are no longer operating.
θ :	Percentage of shops that returned to the use of electronic commerce.
β :	Percentage of shops that did not use e-commerce and now oppose its use.
δ :	Percentage of shops that no longer want to use e-commerce.
η :	Percentage of stores that have temporarily stopped using e-commerce.

Fig. 1: Model parameters

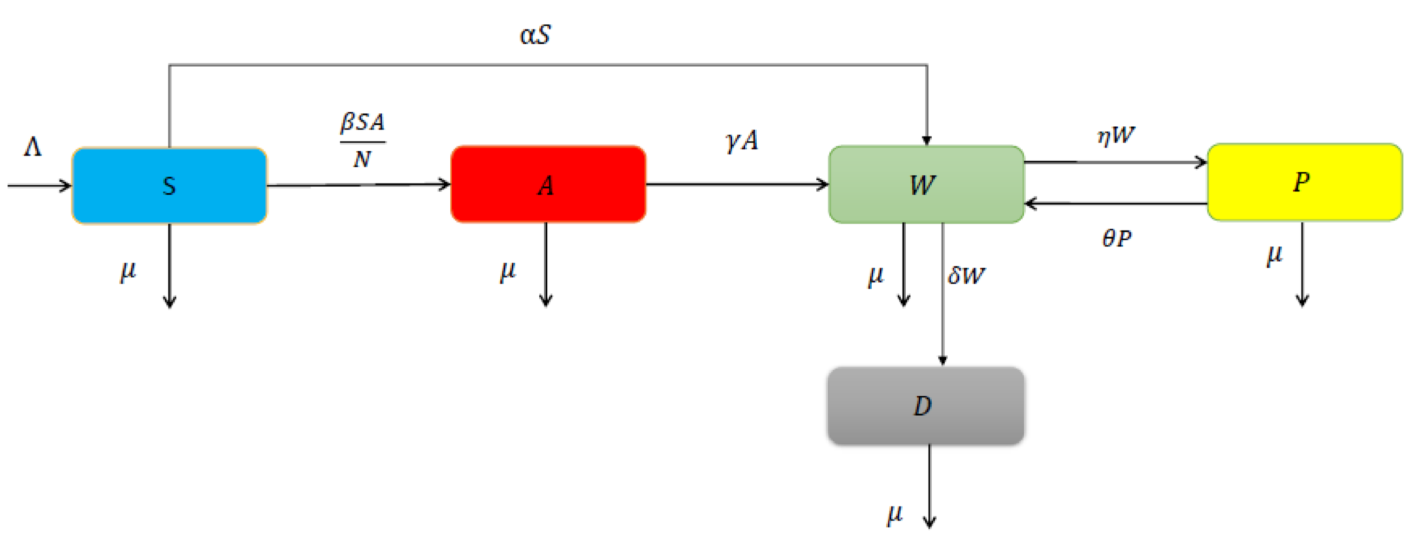


Fig. 2: E-Commerce

Results

Theorem 1 . If we have the initial conditions of the system (1) check $S(0) \geq 0$, $A(0) \geq 0$, $W(0) \geq 0$,$P(0) \geq 0$ and $D(0) \geq 0$ then solutions $S(t)$, $A(t)$, $W(t)$, $P(t)$ and $D(t)$ of system (1) It will be positive for all $t \geq 0$.

lemma 1 . Let the region Ω be defined by:
$\Omega = \left\{ (S(t), A(t), W(t), P(t), D(t)) \in \mathbb{R}^5, S(t) + A(t) + W(t) + P(t) + D(t) \leqslant \frac{\Lambda}{\mu} \right\}$ if we have the initial conditions $S(0), A(0), W(0), P(0)$ and $D(0)$ are positive then the region Ω is positively invariant for the model (1).

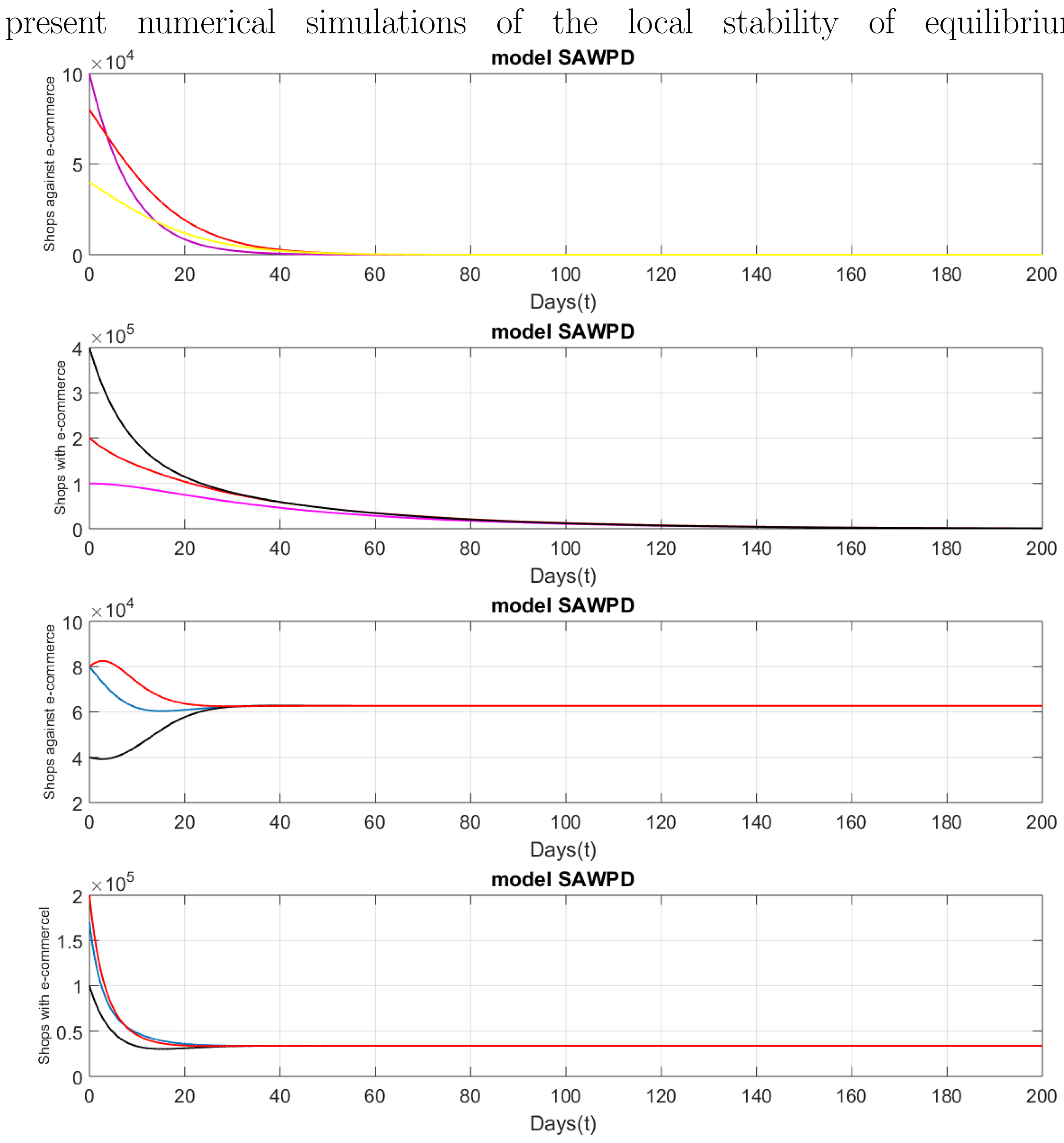
Theorem 2 . The OFEP is locally asymptotically stable if $R_0 < 0$.

Theorem 3 . The OEP is locally asymptotically stable if $R_0 \geq 1$.

We have : $R_0 = \rho\left(FV^{-1}\right) = \beta\frac{\mu}{(\alpha+\mu)(\gamma+\mu)}$.

Numerical simulations

We present numerical simulations of the local stability of equilibrium points,



Conclusion

In summary, in this paper we address the modeling of e-commerce, adopting a stepwise approach inspired by epidemiological dynamics. In order to study the local stability of this model, we calculated R_0 using the Van den Driesch and Watmough method. Finally, we performed numerical simulations in the Matlab application in order to confirm the results obtained.

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Misinformation and disinformation spreading model

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Introduction

This paper delves into the pervasive issue of misinformation and disinformation propagation within social networks. Employing a compartmental model, inspired by epidemiological modeling [1], we characterize the dynamics of information diffusion as it spreads through different segments of a population. The model incorporates distinct compartments representing individuals susceptible to misinformation, actively spreading it, and those who have developed immunity to its influence. To mitigate the detrimental impact of misinformation, we introduce optimal control strategies that dynamically manipulate key parameters influencing the spread of false information. Leveraging control theory, we formulate an optimization problem to minimize the prevalence of misinformation while considering resource constraints and ethical considerations. Our findings highlight the effectiveness of targeted interventions in curtailing the dissemination of misinformation. The proposed compartmental model, coupled with optimal control strategies, provides valuable insights for policymakers and social media platforms seeking evidence-based approaches to counteract the harmful effects of false information in contemporary communication ecosystems [2] [3].

The optimal control problem

In this research work a modified Susceptible Exposed Infected Recovered ($SEIR$) model is proposed, where the population is divided into five compartments, the susceptible class (S_p), the exposed class (E), the ignorant spreaders class (I_s), the scammers class (S_c) and the stifles class (S_t) compartment. The total population at a given time t denoted by $N(t)$ is equal to the sum of the compartments, that is,

$$N(t) = S_p(t) + E(t) + S_c(t) + I(t)s + S_t(t). \tag{1}$$

Therefore, the flow of transmission of false information within groups of people is governed by the following system of differential equations:

$$\begin{aligned}
\frac{dS_p(t)}{dt} &= \pi - \beta S_p(I_s + S_c) - \mu S_p - u(t)S_p,\\
\frac{dE(t)}{dt} &= \beta S_p(I_s + S_c) - \alpha_1 E - \alpha_2 E - \gamma E - \mu E - v(t)E,\\
\frac{dI_s(t)}{dt} &= \alpha_1 E - \tau I_s - m_1 I_s - \mu I_s,\\
\frac{dS_c(t)}{dt} &= \alpha_2 E + \tau I_s - m_2 S_c - \mu S_c - w(t)S_c,\\
\frac{dS_t(t)}{dt} &= \gamma E + m_1 I_s + m_2 S_c - \mu S_t + u(t)S_p + v(t)E + w(t)S_c.
\end{aligned} \tag{2}$$

with the initial conditions

$$S_p(0) \geq 0,\ E(0) \geq 0,\ I_s(0) \geq 0,\ S_c(0) \geq 0,\ S_t(0) \geq 0.$$

The problem is to minimize the objective functional

$$J(u, v, w) = S_c(T) + E(T) - S_t(T) + \int_0^t [S_c(t) + E(t) - S_t(t) + \frac{A}{2}u(t)^2 + \frac{B}{2}v(t)^2 + \frac{C}{2}w(t)^2] \tag{3}$$

Where $A > 0$, $B > 0$ and $C > 0$ are the cost coefficients. They are selected to weigh the relative importance of $u(t)$, $v(t)$ and $w(t)$ at time t, T which is the final time.

S_p	Susceptible group of people at a given time (t)
E	Set of people that are exposed to rumor scam at a time (t)
I_s	The set of people that are ignorantly spreading scam rumor at a given time (t)
S_c	The set of scammers that are deliberately spreading scam rumor at a given time (t)
S_t	Stifler (those who refuse or have stop spreading scam rumor) at a given time (t)

Fig. 1: Model parameters

Parameter	Parameter description
π	Recruitment rate into the susceptible class
β	Rate constant at which the susceptible become exposed to rumor scams
α_1	Rate constant of moving from the exposed class to the ignorant spreaders class
α_2	Rate constant of moving from the exposed class to the scammers class
m_1	Rate constant of moving from the ignorant class to the stiflers class
m_2	Rate constant of moving from the scammers class to the stiflers
τ	Rate constant of moving from the ignorant class scammers class
γ	Rate constant of moving from the exposed class to the stiflers class
μ	Natural death rate of people in any class

Fig. 2: Model parameters

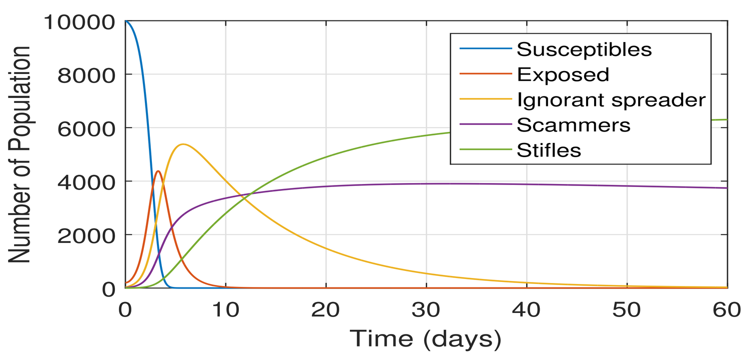


Fig. 3: Dynamics spread of the model

Results

Theorem 1 *Consider the control problem for system (4). There are three optimal controls* $(u^*(t), v^*(t), w^*(t)) \in U^3$ *such that*

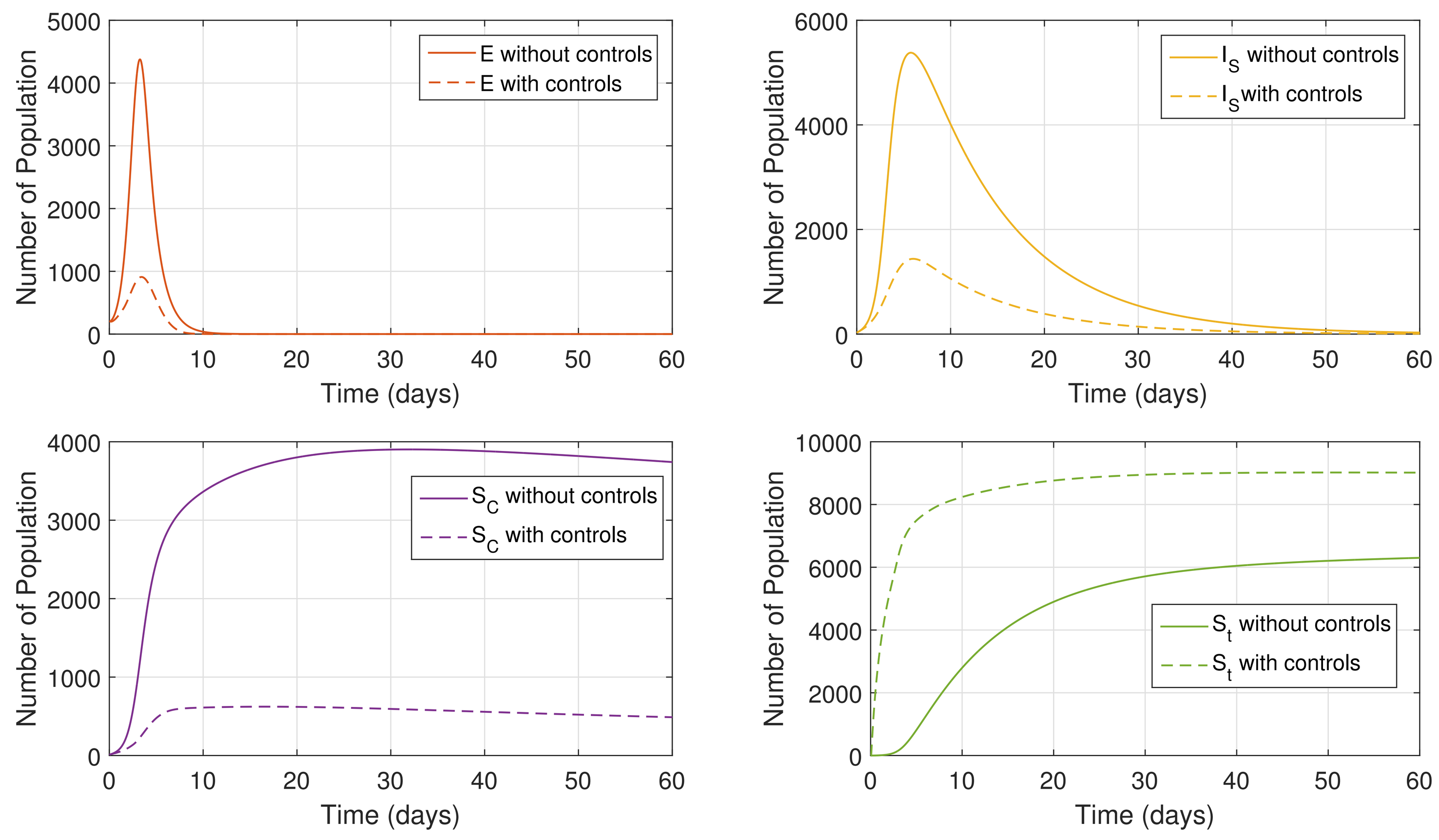
$$J(u^*(t), v^*(t), w^*(t)) = \min_{u,v,w \in U} J(u(t), v(t), w(t))$$

Given the optimal control $(u^*(t), v^*(t), w^*(t))$ *and the solutions* S_p^*, E^*, I_S^*, S_c^* *and* S_t^* *of the corresponding state system (2) there exists adjoin variables* $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ *and* λ_5 *satisfying:*
Such that, the optimal controls $u^*(t)$,$v^*(t)$ *and* $w^*(t)$ *are given by*

$$\begin{aligned}
u^*(t) &= \min\left(1; \max\left(0; \frac{1}{A}(\lambda_1 - \lambda_5)S_p\right)\right);\\
v^*(t) &= \min\left(1; \max\left(0; \frac{1}{B}(\lambda_2 - \lambda_5)E\right)\right);\\
w^*(t) &= \min\left(1; \max\left(0; \frac{1}{C}(\lambda_4 - \lambda_5)S_c\right)\right);
\end{aligned} \tag{4}$$

Numerical simulations

We give the numerical simulations to showcase the effectiveness of our proposed optimal controls $u(t)$, $v(t)$ and $w(t)$,



Conclusion

In summary, this paper has delved into the intricate modeling of misinformation and disinformation spread within social networks, adopting a compartmental approach inspired by epidemiological dynamics. The introduction of three optimal control strategies, guided by Pontryagin's Maximum Principle, offers a sophisticated and adaptable framework for tackling the challenges posed by misinformation.

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Stochastic control model for COVID-19

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Introduction

Over the years, the issue of infectious disease epidemics has persisted as a noteworthy global public health challenge, necessitating extensive collaboration among researchers across various fields. Mathematicians, in particular, contribute significantly by employing modeling techniques and optimization methods. The exploration of infectious disease epidemic modeling is not a recent pursuit; it dates back to the 18th century with Daniel Bernoulli's model addressing smallpox. The utilization of modeling can take either a deterministic or stochastic approach, depending on the characteristics of the system under consideration and the presence or absence of unpredictability or randomness. In turn, optimization plays a pivotal role in understanding the origins of diseases, forecasting their progression, and devising effective strategies to control their spread.

Problematic

To prevent the spread of an infectious disease such as Covid-19, it's imperative to adopt, in first place and before any vaccination, some protective measures such as face masks and active testing/screening. However, the challenge does not lie in adopting these protective measures, but rather in identifying the optimal and effective approach for their implementation. This becomes particularly crucial as some countries have experienced a high number of infections despite having implemented protective measures. Consequently, this prompted us to pose the following questions:

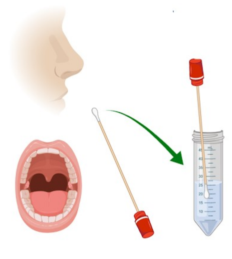
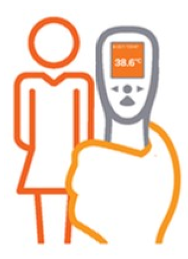
Are these preventative measures effective against infectious diseases?
What is the optimal strategy that minimizes the number of infections?

Our problem can be formulated mathematically such as:

$$\begin{cases} dX(t) = f\left(t, X(t), v(t)\right) dt + \sigma\left(t, X(t)\right) dW(t), \forall\, t \in [0, T], \\ X(0) = X_0 \in \mathbb{R}^4, \end{cases}$$

Where $v = (v_1, v_2)$ are our optimal controls aiming to minimize this cost functional:

$$J(v) = \mathbb{E}\left[\int_0^T g(t, X(t), v(t))dt + \varphi(X(T))\right],$$

Method

1. Stochastic differential equations
2. Stochastic process
3. Optimal control theory
4. Stochastic maximum principle
5. stability analysis

Examples and plots

To demonstrate the efficacy of preventive measures against an infectious disease, we employ the SEIR model, where S denotes susceptible individuals, E represents those exposed to the infection, I indicates infected individuals capable of transmitting the disease, and R designates individuals who have recovered and gained immunity.
Our focus is on the context of Covid-19 in Morocco, and the initial parameter values are derived from publicly available data on confirmed COVID-19 cases in Morocco.

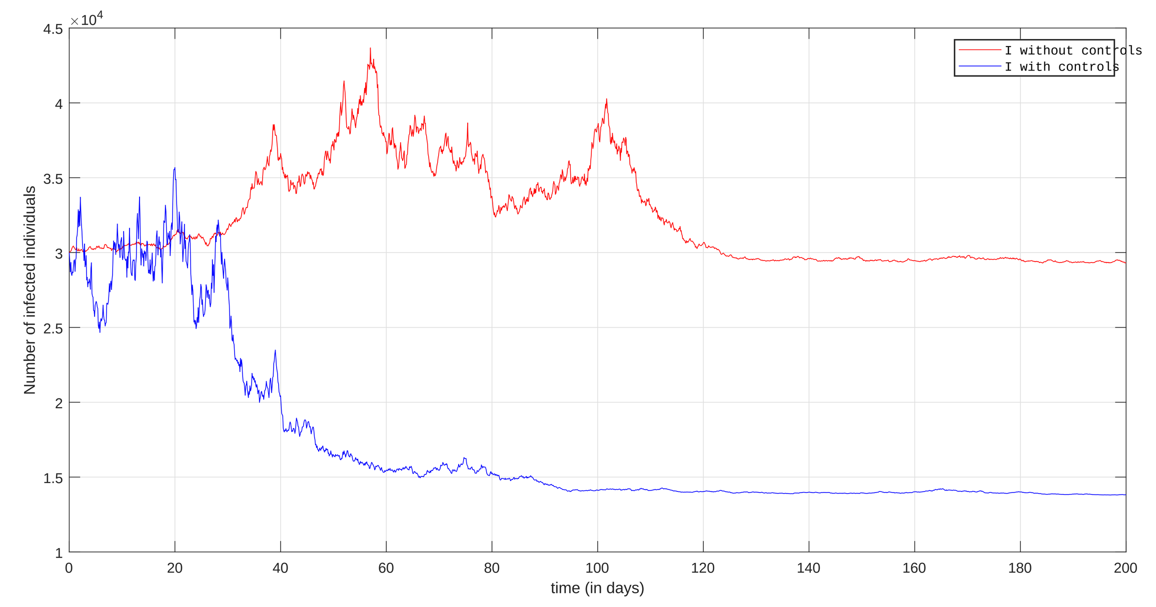


Fig. 2: Estimation of the stochastic infected function in the absence/presence of protective measures

Key Results

1. The use of face masks, along with active screening and testing, commonly considered fundamental measures, has proven effective in significantly reducing the numbers of both infected and exposed individuals.
2. The minimization of infected and exposed individuals holds true even in scenarios where the disease had already begun to spread.
3. Prioritizing public health measures slows the spread of the disease.

Conclusion and future Work

The onset of the transmission of infectious diseases occurs at the level of individual interactions within a population, particularly when vaccination is not in place. In such scenarios, the rate of disease spread is directly influenced by the collective commitment of individuals to implementing preventive measures. The more people actively engage in and adhere to these precautionary actions, the more effective the containment of the disease becomes, resulting in a slower rate of transmission throughout the population.
In essence, the cooperation and dedication of individuals to preventive measures play a pivotal role in mitigating the rapid dissemination of infectious diseases.

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